Gauss-Jordan Elimination

Row Echelon form $\begin{bmatrix}1 & x\\0 & 1\end{bmatrix}$

Reduced row echelon form $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Back Substitution: put into R.E. form, then solve from the bottom, upward.

Rank(A) = number of leading 1's in row echelon form

Homogeneous systems with m < n have non-trivial solutions

Matrix operations

Transposition: rows become columns, col's become rows

Trace (tr): the sum of the numbers on the diagonal (n x n)

Like rotating the page about the diagonal line

Transformation (composition): say, from X to Y to Z means:

Multiplication: Z(YX) (seemingly backwards)

Representing system as combination of matrices:

$6x + y = 0$

$4x - 3y = -2$

Same as:

$A\vec{x} = \vec{b}$

$$\begin{bmatrix}6 & 1\\4 & -3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\-2\end{bmatrix}$$

Matrix Properties & Non-Properties

1) $A0=0$
2) But $A\neq 0$ and $B\neq 0$ does not imply that $AB\neq 0$
3) $AB=AC$ and $A\neq 0$ does not always imply that $B=C$
4) AB not usually equal to BA
5) $(AB)C=A(BC)$ [Associative]
6) $I = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
7) $A(B+C)=(AB)+(AC)$
8) $0+A=A$
9) $1A=A$
10) $(k+m)A=kA+mA$
11) $kmA=k(mA)$
12) $k(AB)=(kA)B=A(kB)$
13) $(A+B)^T=A^T+B^T$
14) $(kA)^T = k(A)^T$
15) $(AB)^T = B^T A^T (\neq A^T B^T)$
16) $(AB)^{-1} = B^{-1} A^{-1}$

Matrix inverse

Formula:

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}^{-1} = \frac{1}{ad-cb}\cdot\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$$ Assumes ad - cb $\neq 0$

Elementary row method:

Just tag on 'I' on the right side of the A.M.:

$\begin{bmatrix}a & b & 1 & 0\\c & d & 0 & 1\end{bmatrix}$ And follow the usual GJ method to solve

Big Theorem

A^{-1} exists...

$\Leftrightarrow A\vec{x} = \vec{0}$ has only trivial solutions

$\Leftrightarrow Rank(A) = n$ (A ~ I via G-J)

$\Leftrightarrow \forall \vec{b}\in R^n, A\vec{x}=\vec{b}$ has ≥ 1 solutions

$\Leftrightarrow \forall \vec{b}\in R^n, A\vec{x}=\vec{b}$ has ≤ 1 solutions

$\Leftrightarrow \forall \vec{b}\in R^n, A\vec{x}=\vec{b}$ has $=1$ solutions

$\Leftrightarrow \exists \vec{b}\in R^n, A\vec{x}=\vec{b}$ has ≥ 1 solutions

Special Types of Matrices

Symmetric: $A = A^T$

Symmetric Skew: $A = -A^T$

Diagonal: $\begin{bmatrix}c & 0 & 0\\0 & c & 0\\0 & 0 & c\end{bmatrix}$

$\{\text{Diag}\}\cdot A \rightarrow$ scales rows

$A\cdot\{\text{Diag}\} \rightarrow$ scales cols

Upper triangle: $\begin{bmatrix}c & c\\0 & c\end{bmatrix}$

Lower triangle: $\begin{bmatrix}c & 0\\c & c\end{bmatrix}$

Complexity

1) $1+2+3+...+n = \dfrac{n(n+1)}{2}$

2) $1^2+2^2+3^2+...+n^2 = \dfrac{n(n+1)(2n+1)}{6} = \dfrac{n^3}{3}+\dfrac{n^2}{2}+\dfrac{n}{6}$

Determinants

Remember to take ABS value for areas/volumes!!

1) Det (2x2) = ad - bc
2) $Det(I) = 1$
3) $Det(0) = 1$
4) $Det(A') = -Det(A)$, where A' has swapped a row or column
5) $Det(A^T) = Det(A)$
6) Ordinary Row Ops (R2:=R2 + kR1) result in no change
7) $\begin{vmatrix}ka_{11} & ka_{12}\\a_{21} & a_{22}\end{vmatrix} = k\begin{vmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{vmatrix}$

Scaling rows -> multiply the Det(A) by the inverse of the scaling factor

8) $\begin{vmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{vmatrix} + \begin{vmatrix}a_{11} & a_{12}\\b_{21} & b_{22}\end{vmatrix} = \begin{vmatrix}a_{11} & a_{12}\\a_{21}+b_{21} & a_{22}+b_{22}\end{vmatrix}$

Same holds for columns

9) Det (triangular matrix) = $a_{11}\cdot a_{22}\cdot q_{33}\cdot\ldots\cdot a_{nn}$