MTH110 Summary Sheet Jeff Mair

2.1 Logical Equivalence Laws:

1. Commutative law $p \wedge q \equiv q \wedge p$
2. Associative law $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
3. Distributive law $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
4. Identity laws $p \wedge \boldsymbol{t} \equiv p$ $p \vee \boldsymbol{c} \equiv p$
5. Negation law $p \vee \sim p \equiv \boldsymbol{t}$ $p \wedge \sim p \equiv \boldsymbol{c}$
6. Double negative law ~(~p) ≡ p
7. Idempotent law $p \wedge p \equiv p$ $p \vee p \equiv p$
8. Universal bound law $p \vee t \equiv \boldsymbol{t}$ $p \wedge c \equiv \boldsymbol{c}$
9. De Morgan's Law $\sim(p \wedge q) \equiv \sim p \vee \sim q$
10. **Absorption law** $p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$
11. Negations of t and c: $\sim \boldsymbol{t} \equiv \boldsymbol{c}$ $\sim \boldsymbol{c} \equiv \boldsymbol{t}$

2.2 Conditional Statements

Logical equivalence: $p \rightarrow q \equiv \sim p \vee q$

Negation: $\sim(p \rightarrow q) \equiv p \wedge \sim q$

Only-if, is contrapos to usual If-then

Contrapositive: (switch direction, negate both)

Converse: (switch direction)

Inverse: (negate both)

r is Necessary for s: if ~r then ~s. r is Sufficient for s: if r then s

2.3 Valid and invalid arguments

CR is where all premises are True

If there is a critical row with false conclusion then argument invalid

Modus Ponens: If P then Q. P, so Q.

Modus Tollens: If P then Q. Not Q so Not P.

Other valid argument forms:

Generalization: $p.$ $\therefore p \vee q.$

Specialization: $p \vee q.$ $\therefore q.$ and $p \vee q.$ $\therefore p.$

Elimination: $p \vee q.$ $\sim q.$ $\therefore p.$

Transivity: $p \rightarrow q.$ $q \rightarrow r.$ $\therefore p \rightarrow r.$

Proof by division into cases: $p \vee q.$ $p \rightarrow r.$ $q \rightarrow r.$ $\therefore r$

Falacies:

Converse error:Wrong result of M.P.,Inverse error:Wrong result of M.T.

Knights, Knaves

2.4 Digital Logic Ccts

Half-adder: Add two columns, result is Carry,Sum

2.5 Binary computation

Negative 8 bit binary: flip the bits, add 1

3 Predicates, Quantified Statements

Law of Negation of quantified conditional statements: reverse quantifier, negate predicate

6.1 Set theory

Power set is a set of all subsets including null

N elements in a Power Set is 2^n where n is the number of elements in the original set.

Disjoint = no common elements between sets

Element method of proof

Subsets

Theorem 6.2.1 Subset Relations

1. Inclusion of Intersection: $A \cap B \subseteq A$ and $A \cap B \subseteq B$
2. Inclusion in Union: $A \subseteq A \cup B$ and $B \subseteq A \cup B$
3. Transitive property of subsets: if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

Procedural versions of set definitions

1. $x \in X \cup Y \Leftrightarrow x \in X$ or $x \in Y$
2. $x \in X \cap Y \Leftrightarrow x \in X$ and $x \in Y$
3. $x \in X - Y \Leftrightarrow x \in X$ and $x \notin Y$
4. $x \in X^c \Leftrightarrow x \notin X$
5. $(x, y) \in X \times Y \Leftrightarrow x \in X$ and $y \in Y$

Set Identities:

1. Commutative Laws
2. Associative Laws
3. Distributive Laws
4. Identity Laws: a) $A \cup \emptyset = A$ and b) $A \cap U = A$
5. Complement Laws: a) $A \cup A^C = U$ and b) $A \cap A^C = \emptyset$
6. Double Complement Laws
7. Idempotent Laws
8. Universal bound Laws: a) $A \cup U = U$ and b) $A \cap \emptyset = \emptyset$
9. De Morgan's Laws
10. Absorption Laws
11. Complements of U and emptyset a) $U^c = \emptyset$ and b) $\emptyset^c = U$
12. Set Difference Law: $A - B = A \cap B^C$

Element Proof Technique

1. Setup left side as subset of right side
2. Suppose $x \in$ left side
3. Manipulate the left side, into cases to satisfy all of the ∪ conditions cases on the right side.
   Hence...

4 - Direct Proof

The set of integers is closed under addition, multiplication, subtr.

2k + 1 (where k is any integer), is a negative integer

2m (where m is any integer) is a positive integer

4.4 Quotient Remainder Theorem:

$n = dq + r$ and $0 \leq r < d$

4.5 - Floor/Ceiling

n div $d = \lfloor n/d \rfloor$

n mod $d = n - d\lfloor n/d \rfloor$

8.3 Equivalence Relations

Relation induced by a partition:

$x\,R\,y \Leftrightarrow$ both x and y are in the same partition section

Equivalence class:

$[a] = \{x \in A | x\,R\,a\} =$ {equivalence class of a}

$m \equiv n (mod\, d) \Leftrightarrow d\,|(m - n)$